The Game of Life...Mathematics Style

I. UNIT OVERVIEW & PURPOSE:

While discussing graduation and prom plans with a group of juniors and seniors, it was determined that students were going to spend between five hundred to five thousand dollars for these two events. The teacher posed the question "Who is paying for all of this?" The students responded that their moms, dads, and/or grandparents were providing the funds. The next question that the teacher asked was "Do you plan on paying them back?" The majority of the students replied "NO, it is not my responsibility" and a few of them said they would get jobs to pay their own expenses.

The comments from the students caused the teacher to realize that these students are not financially responsible. This project is a direct result of the conversation with the students and the realization that they needed to experience the process of making plans based on an established budget. There was a board game, "Life", that was designed to make the players think about family size, income, living spaces, buying cars to accommodate increased family sizes, expenses, career paths, adjusting for the unexpected and how to handle situations that adults experience in everyday life. The game allowed the players to receive pay checks of certain amounts to help them to develop a budget.

II. UNIT AUTHOR:

Vonna Falls, Monacan High School, Chesterfield County Schools Alisa Cartledge, Thomas Jefferson High School, Richmond Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Algebra

V. OBJECTIVES:

Students will use information related to employment and personal finance to develop a budget for their "family" and link the mathematical data to equations of a line.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE. 12 Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

MPE. 13 Investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

MPE. 16 Investigate and analyze functions algebraically and graphically.

MPE. 19 Graph linear equations and linear inequalities in two variables.

- A.6 The student will graph linear equations and linear inequalities in two variables, including
- a) determining the slope of a line when given an equation of the line, the graph of

the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and

- b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.
- A.8 The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.
- AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include
- a) domain and range, including limited and discontinuous domains and ranges;
- b) zeros;
- c) x- and y-intercepts;
- d) intervals in which a function is increasing or decreasing;
- e) asymptotes;
- f) end behavior;
- g) inverse of a function; and
- h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

NCTM

Algebra

Understand patterns, relations, and functions

Grades 9–12 Expectations: In grades 9–12 all students should-

- generalize patterns using explicitly defined and recursively defined functions;
- understand relations and functions and select, convert flexibly among, and use various representations for them;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- interpret representations of functions of two variables

Use mathematical models to represent and understand quantitative relationships

draw reasonable conclusions about a situation being modeled.

Analyze change in various contexts

• approximate and interpret rates of change from graphical and numerical data.

Process Standards

Problem Solving

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Communication

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

VII. CONTENT:

In this unit, students will review various types of linear equations while working in groups to develop a "family" budget and produce graphs based upon that information. This unit connects with personal finance and economics.

VIII. REFERENCE/RESOURCE MATERIALS:

Will vary for each lesson but will include: paper, pencil, calculators, rulers, print materials and/or the internet for employment and expense research, Excel and other technologies for student presentations.

IX. PRIMARY ASSESSMENT STRATEGIES:

Students will use algebraic concepts to derive the equations of lines by using real life examples of budgets. Each activity will have its own worksheet or project that will be graded using a rubric.

X. EVALUATION CRITERIA:

Students will have rubrics for lessons requiring mathematical computations.

XI. INSTRUCTIONAL TIME:

2-4 days for 90 minute classes or 3-5 days for 45 minute classes. Depending upon the depth of the lessons, ie teacher and student doing extension activities in class or for homework activities may take longer than a class period to complete. Therefore lessons will be listed as "activities" and maybe combined as necessary by the teacher.

Activity 1

Mathematical Objective(s)

In this lesson students review some common linear equations and their graphs. Students will then be asked to relate the graphs to jobs.

Materials/Resources

No special materials are necessary for this activity. Pencils and pens. Graph paper and graphing calculators are optional if students want to check their written linear equations.

Activity 1

Students are given an activity sheet and are asked to review three graphs. (See Activity 1 Handout) They should recognize the labels on the axes. Working individually students should answer the three questions related to the graphs. After an appropriate amount of time, students should then be allowed to work with a neighbor and compare answers. They should take notes on what their partner's answers. The teacher should monitor student interactions and identify groups or individuals that can present their ideas. Students should then be given time to present their findings to the whole group. A class discussion should be held based upon their answers. At the end of the activity students should submit their answers along with the notes they took with their partner.

Answers.

1) Constant function. Salaried employee. The number of hours worked does not influence wages.

Direct variation. Hourly employee. The amount of wages directly reflects the number of hours worked.

Hourly employee that has an initial salary but varies afterwards.

- 2) Answers for this activity may vary. It is up to the teacher to determine if the student has submitted an appropriate equation.
- 3) Answers for this activity may vary. It is up to the teacher and students together to determine if the student has submitted an appropriate job/career that models the particular graph.

Extensions and Connections

Depending upon the length of the previous discussion the extension question may be given during class or given as a homework assignment. (See Extension Questions Activity 1)

Answers

- 1) There should be no negative values for the x or y axes (unless the teacher wants to discuss the possibility of an employee receiving an "advance" on a paycheck and must work to pay it off without receiving any current funds)
- 2) Answers may vary.
- 3) The graph should look like a piecewise function.
- 4) Answers may vary. It is up to the teacher and students to determine if this scenario exists.

Name:	_
Partner's Name	
Activity 1 Handout	
Consider the following graphs of I	near functions.
WAGES *	WAGES †
+ Hot	HOURS HOURS
1. Describe the situation mo	deled by each graph.
2. Write an equation that m	odels each graph.
3. Think about a job/career	hat would relate to the graphs. Explain your answers.
Notes from partner	

Extension Questions Activity 1
Name
Partner's Name
Now that you have had an opportunity to discuss the graphs, equations and possible jobs/careers that maybe associated with them, take a few moments to think about the following questions and answer them accordingly. You may discuss your answers with one other person.
 In the examples from class, would the entire graph exist in the real world? In other words, does the graph of hours and wages make sense over its entire domain and/or range? Explain your reasoning. Give examples.
2. What would the graph of a job in which a person makes straight commission look like? Is it linear?
3. You have a job in which you are paid a salary of \$300 until you work 40 hours when you are granted an overtime rate of \$25 per hour. Describe and sketch the graph.
4. Analyze the graph that you just created. State the domain, range, solution(s), and x-intercept(s).
5. Can you think of a job scenario where a person starts out with more money than they end up with? What would this graph look like?

Activity 2

Mathematical Objective(s)

Students will create a monthly budget according to Income, Family Situation, and Surprises (which could be unexpected debt or inheritance.)

The budget will include:

- a) Rent/Mortgage
- b) Utilities (Power, Water, Gas, Phone, and Trash Collection if applicable)
- c) Transportation (Gas money, Tolls, Parking Meters, and/or Bus Fare)
- d) Food/Personal items (Specific prices and quantity)
- e) Extra Expenses (Cell phones, Entertainment, Clothing, Credit Card(s), Doctors visits, Eating Out, etc.)

Materials/Resources

Pencils, pens, apartment guides/newspaper/ internet for research on home prices and living costs. Excel is needed for the extension activity. Graphing calculators are optional.

Activity 2 Part 1 "Game of Life" and Financial Worksheet

Teacher Directions:

- 1) Print out Income, Family Situation, and Surprise Cards. Then FOLD the cards and LAMINATE them before use.
- 2) Separate students into groups. Assign roles in each group (Recorder, Researcher, Materials Manager, and Reporter)
- 3) A sample of cards is included with this unit. A short list of jobs and their salaries are also included so that the teacher may incorporate different salaries as they may deem necessary. Additional jobs and approximate salaries maybe accessed through http://www.careercast.com/jobs-rated/2012-ranking-200-jobs-best-worst

Student Directions:

- 1) Place cards in three stacks on a table and Instruct the Materials Manager get one card out of each stack.
- 2) Then the Recorder will write down the information from each card as the other members begin devising a plan of action. (The Materials Manager may have to get an extra income card depending on the family situation.)
- 3) The Materials Manager will gather newspapers, magazines, apartment/home guide, etc.
- 4) The Researcher will begin pricing housing, utilities, and transportation while the Recorder writes down the information. (If technology is available, the research can be done online.)

5) The Materials Manager and the Reporter will continue working on the financial plan of action.

Assessment

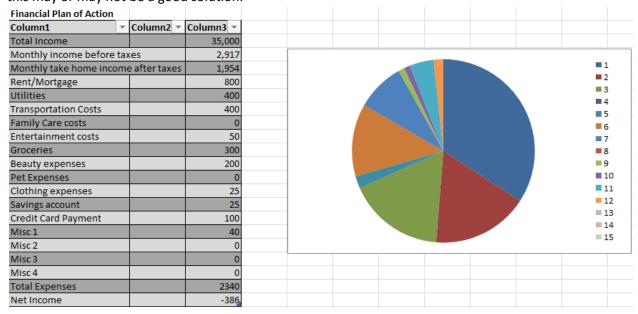
- 1) The teacher will conduct an informal assessment of Activity 2 and listen to group discussions about the financial plan.
- 2) Students will answer the following question with the information they have collected up to this point in their research and turn it in for review. (See Assessment Activity 2)

Activity 2 Part 2 Excel Spreadsheet

Teacher Directions:

1) Review the example with the students and facilitate a quick discussion about the problem with the Monthly take home income and the Total Expenses.

Students may suggest getting a roommate, finding a cheaper place to live, getting a friend to take over beauty expenses, forgoing entertainment, and/or going home to eat with their parents to lessen the grocery bill. Some students may even suggest using the credit card to "make ends meet". Discuss why this may or may not be a good solution.



2) Have students create an Excel spreadsheet and pie graph to gain a better understanding and visual representation of where their money is being spent per month.

Student Directions:

Create an Excel spreadsheet to model the information in your Financial Plan of Action. Use formulas to calculate the Monthly income before taxes and after taxes. Using the rows as each expense, make a pie chart for a visual representation of your expenses. Make sure to sum up your total expenses. This will help you gain a better understanding of where your money is being spent and will let you know if any adjustments could to be made.

Game Cards

INCOME CARD

Software Engineer \$88, 142.00

Extension: Write a possible equation that models this income.

FAMILY SITUATION CARD

Single Income household Two children One pet

SURPRISE CARD

Family inheritance of \$5,000

INCOME CARD

Dental Hygienist \$68,109.00

Extension: Write a possible equation that models this income.

FAMILY SITUATION CARD

Double income household Two children One pet

SURPRISE CARD

Pay the Internal Revenue Service \$2500

INCOME CARD

Dietician \$53,127.00

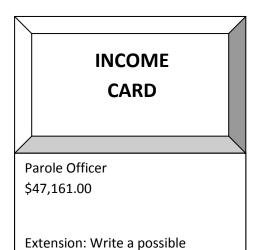
Extension: Write a possible equation that models this income.

FAMILY SITUATION CARD

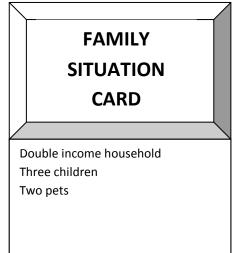
Single income household No children No pets

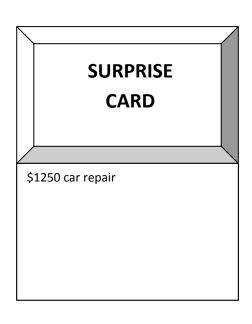
SURPRISE CARD

\$500 Visa gift card



equation that models this income.







Surgeon \$305,078.00

Extension: Write a possible equation that models this income.

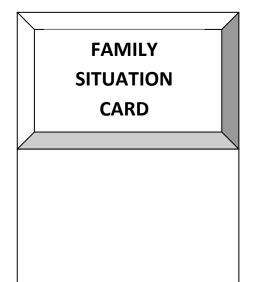
FAMILY SITUATION CARD

Single Income household One Child No pets

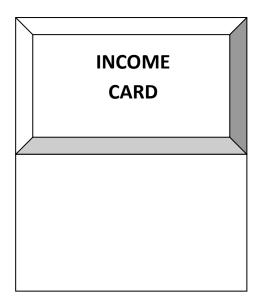
SURPRISE CARD

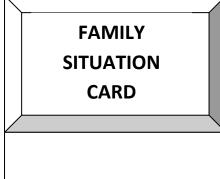
You forgot to pay your power bill and now you have a late charge of \$35 added to next month's bill.

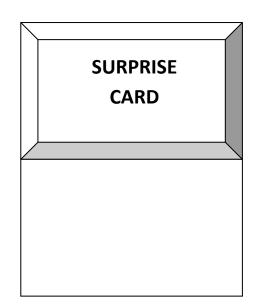
INCOME CARD

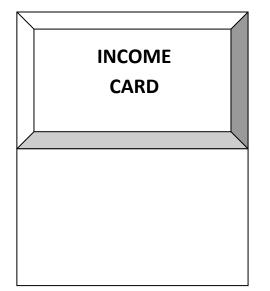


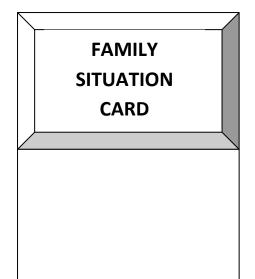
SURPRISE CARD

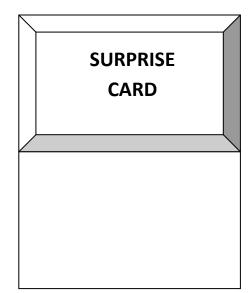


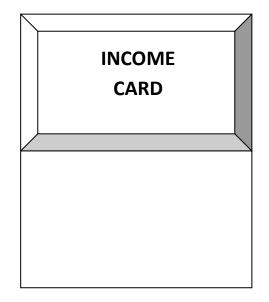


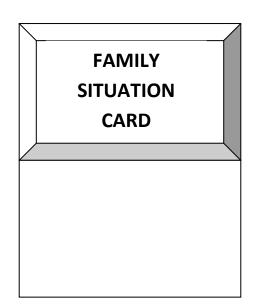


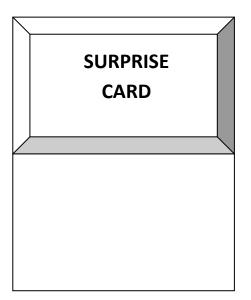




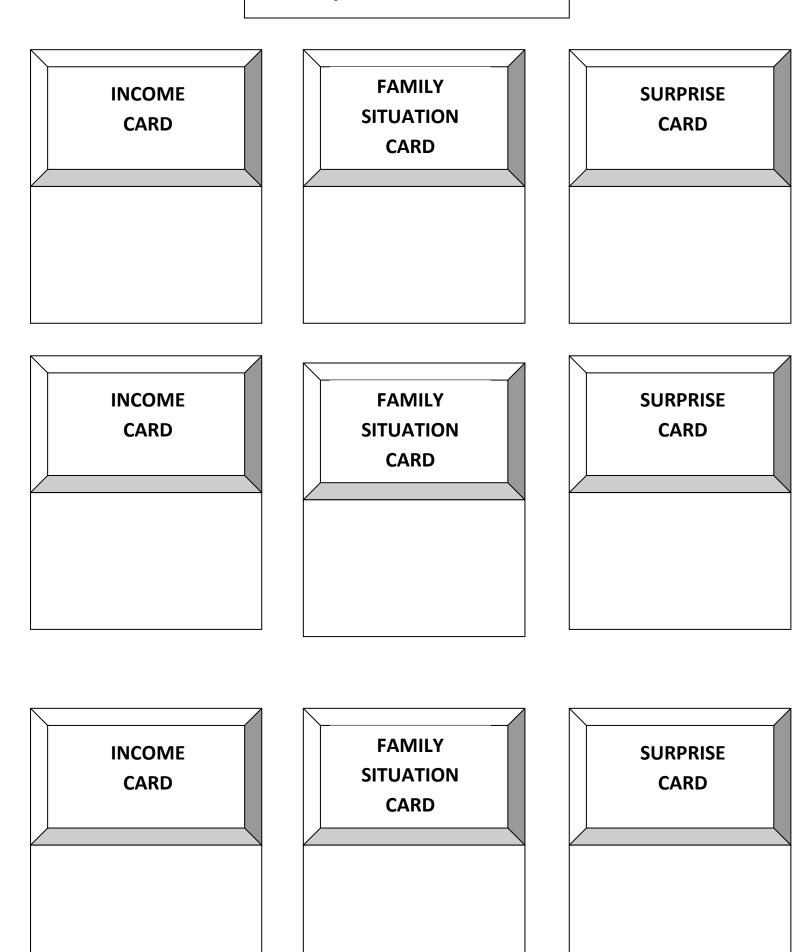








Blank/Extra Game Cards



Name		 		
Group	Members			

Financial Plan of Action

Using the job/career you have selected and the research you and your group have completed, fill in the following categories to determine your net income.

	Amount
Take Home Income (Enter the amount of pay per month minus taxes approximately 33%)	
Monthly Expenses	
Rent/Mortgage	
Utilities (These include water, electric, sewer, natural gas, propane or other ways in which to heat the home or provide running water)	
Transportation Costs (These include but may not be limited to: gas, car insurance, personal property taxes, routine car maintenance or public transportation costs)	
Family Care Costs (These may include daycare expenses or nursing home expenses for elderly parents)	
Entertainment Costs (Dinner out, movies, cable or satellite etc.)	
Groceries	
Beauty Expenses (Hair, nails, etc)	
Pet Expenses (This includes food, grooming, pet sitting or walking)	
Clothing Expenses	
Savings Account	
Credit Card Payments	
Miscellaneous Expenses (Other expenses your group finds necessary to include)	
1)	
2)	
3)	
4)	
Total Expenses	
Net Income (Take Home Pay minus total expenses)	

Top Jobs from 2012

Adapted from http://www.careercast.com/jobs-rated/2012-ranking-200-jobs-best-worst

Software Engineer

Researches, designs, develops and maintains software systems along with hardware development for medical, scientific, and industrial purposes.

Income: \$88,142.00

Human Resources Manager

Plans, directs, and/or coordinates all human resource activities and staff of an organization.

Income: \$99,102.00

Dental Hygienist

Assists dentists in diagnostic and therapeutic aspects of a group or private dental practice.

Income: \$68,109.00

Financial Planner

Related to careers in portfolio management, the financial planner offers a broad range of services aimed at assisting individuals in managing and planning their financial future.

Income: \$104,161.00

Computer Systems Analyst

Plans and develops computer systems for businesses and scientific institutions.

Income: \$78,148.00

Mathematician

Applies mathematical theories and formulas to teach or solve problems in a business, educational, or industrial climate.

Income: \$99,191.00

Optometrist

Diagnoses visual disorders and prescribes and administers corrective and rehabilitative treatments.

Income: \$95,152.00

Physical Therapist

Plans and directs treatment to improve mobility and alleviate pain in persons disabled by injury or disease.

Income: \$76,100.00

Pharmacist

Advises physicians and patients on the affects of drugs and medications; prepares and dispenses prescriptions.

Income: \$112,070.00

Web Developer

Creating and maintaining layout, navigation, and interactivity of intranet and internet websites.

Income: \$76,179.00

Veterinarian

Administers to the care of animals through the use of preventative and diagnostic techniques.

Income: \$82,190.00

<u>Physicist</u>

Researches and develops theories concerning the physical forces of nature.

Income: \$106,183.00

Civil Engineer

Plans and supervises the building of roads, bridges, tunnels, and buildings.

Income: \$78,133.00

Dentist

Examines, cleans, and repairs teeth, and diagnoses and treats diseases and abnormalities of the mouth.

Income: \$141,144.00

Computer Programmer

Organizes and lists the instructions for computers to process data and solve problems in logical order.

Income: \$71,178.00

Nurse (Registered)

Assists physicians in administering holistic medical care and treatment to assigned patients in clinics, hospitals, public health centers, and health maintenance organizations.

Income: \$65,116.00

Physician (General Practice)

Performs examinations, diagnoses medical conditions, and prescribes treatment for individuals suffering from injury,

discomfort or disease. Income: \$205,029.00

Psychologist

Studies human behavior, emotion, and mental processes, and provides counseling and therapy for individuals.

Income: \$67,179.00

Psychiatrist

Studies, diagnoses, and treats mental, emotional and behavioral disorders.

Income: \$164,250.00

Optician

Fills lens prescriptions, and fits eyeglasses and contact lenses.

Income: \$33,143.00

Accountant

Prepares and analyzes financial reports to assist managers in business, industry and government.

Income: \$62,174.00

Social Worker

Assists individuals, families, and groups in need of counseling and special social services.

Income: \$40,162.00

Aerospace Engineer

Designs, develops, and tests new technologies concerned with the manufacture of commercial and military aircraft and

spacecraft.

Income: \$97,134.00

<u>Judge</u>

Arbitrates legal matters coming under the jurisdiction of the federal government, using a thorough knowledge of federal statutes and legal precedent.

Income: \$119,429.00

Heating/Refrigeration Mechanic

Installs and services air-conditioning and furnace systems in businesses and residences.

Income: \$43,158.00

Plumber

Builds and repairs water, waste disposal, drainage and gas delivery systems for residential, commercial and industrial

structures.

Income: \$47,186.00

Bookkeeper

Maintains financial records and prepares statements of a company's income and daily operating expenses.

Income: \$34,143.00

Electrician

Maps layout and installs and repairs electrical wiring and fixtures.

Income: \$48,179.00

Receptionist

Greets visitors to offices, answers questions, and refers customers to appropriate staff.

Income: \$25,106.00

Attorney

Counsels clients in legal matters; using interpretation of laws and rulings to advise and represent businesses and individuals.

Income: \$113,219.00

Vending Machine Repairer

Performs maintenance and repairs on coin-operated vending and amusement machines.

Income: \$30,147.00

Funeral Director

Prepares bodies for burial, and arranges and directs funerals.

Income: \$54,230.00

Clergy

Leads a congregation in worship and other spiritual services, provides moral guidance to members, and participates in

community outreach. Income: \$44,221.00

Machine Tool Operator

Operates computerized machines in the manufacture of industrial parts.

Income: \$35,122.00

Cosmetologist

Creates hair styles, and advises clients about caring for their hair between appointments.

Income: \$23,163.00

Security Guard

Protects property from damages incurred by theft, fire, and vandalism.

Income: \$24,141.00

120. Emergency Medical Technician

Attends to situations which demand immediate medical attention, such as automobile accidents, heart attacks, and gunshot

wounds.

Income: \$30,155.00

Corrections Officer

Supervises inmates' activities and enforces regulations in jails, prisons, and other correctional facilities.

Income: \$39,158.00

Bus Driver

Transports passengers according to a specific schedule along metropolitan and community routes.

Income: \$36,171.00

Agricultural Scientist

Researches methods to improve quantity and quality of yields from farm crops and livestock, and attempts to find practical solutions to problems in agriculture.

Income: \$33,141.00

Salesperson (Retail)

Provides courteous and efficient service to customers in retail stores.

Income: \$21,144.00

Recreation Worker

Organizes and supervises a variety of leisure activities, including sports, arts, crafts, drama, singing, dancing, and story telling.

Income: \$22,138.00

<u>Janitor</u>

Cleans offices and other spaces within buildings, and keeps areas in good condition.

Income: \$22,131.00

Highway Patrol Officer

Patrols roads and highways, and enforces traffic regulations and criminal statutes.

Income: \$54,163.00

Cashier

Receives payments, makes change, and provides receipts for goods sold.

Income: \$19,063.00

Child Care Worker

Cares for infants and toddlers when parents are at work or are unable to do so for other reasons.

Income: \$19,081.00

Garbage Collector

Collects refuse on a designated municipal route, and transports trash to disposal plants or landfill areas.

Income: \$33,184.00

Police Officer

Provides protection against crime, investigates criminal activity, and works with the public on crime-prevention measures.

Income: \$54,163.00

Bank Teller

Cashes checks, makes deposits and withdrawals, and handles a variety of other transactions for bank customers.

Income: \$24,083.00

<u>Farmer</u>

Manages the successful operation of a crop, livestock, dairy, or poultry farm.

Income: \$61,269.00

Disc Jockey

Selects and plays records or tapes; comments on areas of interest to a particular radio audience.

Income: \$27,387.00

Na	me:
	nat percent of your paycheck (monthly take home pay after taxes) is ng to each item/expense in your Financial plan?
	ny would knowing these percentages be important in financial decision king?
ı	

Activity 3

Mathematical Objective(s)

Students will write the equation of a line from a given scenario and graph it using an online applet.

Materials/Resources

Paper, pencils, computer with internet capabilities and graphing calculators (optional)

Activity 3

Warm Up (See Warm Up Activity 3). Answers follow the activity.

Teacher Directions:

- 1) Facilitate a discussion about income versus graph model.
- 2) Assist students with decision making.

Possible Websites for students to reference:

http://www.ncsl.org/issues-research/labor/state-minimum-wage-chart.aspx

http://en.wikipedia.org/wiki/Minimum wage in the United States

http://www.infoplease.com/ipa/A0774473.html

http://www.infoplease.com/homework/math-money.html

http://www.mathsisfun.com/equation of line.html

http://www.mathwarehouse.com/algebra/linear_equation/write-equation/equation-of-line-given-two-points.php

http://www.classzone.com/eservices/home/pdf/student/LA202GAD.pdf (see example 6)

Student Directions:

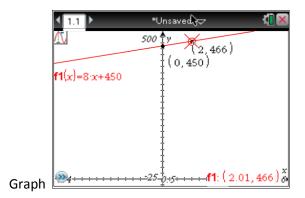
- 1) From Activity 1, decide which graph would best model your income (from Activity 2) and explain why.
- 2) Use the information from your income card and write the equation of the line.
- 3) Continue making your budget in your groups. Finalize your research to prepare for your group presentation.

Warm-Up Activity 3

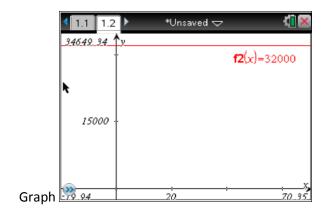
Name:	
Date:	-
recruits at most 3 new custor make during the course of a	at Company A plus \$8 for every new customer he recruits. Homers per week. What is the least amount of money Samuel camonth? What is the greatest amount? Why do you think this muel? Sketch a graph that models the amount of money that.
sometimes she works 60 hou	ry year. Sometimes she works 40 hours per week and Irs per week. If Tabitha is a salaried employee, sketch a graph then write an equation that represents how much money sho
3) Are the graphs different?	Explain.

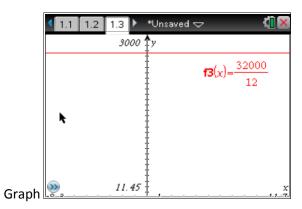
Answers:

1) The least amount of money that Samuel can make is \$1800. The most that he may make is \$1896. This information may be important to him so that he can set up a monthly budget for himself. Any extra money that he earns could go into a savings account.



2)





3) Varying Answers.

Activity 4

Mathematical Objective(s)

Student will present their work from the last two days on equations and budgets.

Materials/Resources

No special resources are necessary. Students will need paper, pencils for the peer review and a projection system to view their work.

Activity 4

Teacher Directions:

- 1) Facilitate Presentations
- 2) Wrap-Up discussion. "Which groups' financial situation was the best? Explain your reasoning?"

Student Directions:

- 1) The Reporter from each group will lead the group presentation.
- 2) All other students will complete a Peer Review form.

Assessment

Project Grading Rubric and Peer Review Rubric

Budget Project Grading Rubric

- 3- Exceeds Expectations,
- 2- Meets Expectations,
- 1- Below Expectations, and
- 0 No Attempt is made.

Follows Financial Plan/Stays within budget	3	2	1	0
Matches Income to feasible graph model	з	2	1	0
Neat and Organized	3	2	1	0
Speaks clearly	3	2	1	0
Answers questions thoughtfully	3	2	1	0

Budget Peer Review Form Your Na	ame Group Name	
1) What did you learn from this presentation?		
2) What was your favorite part about this presentation?		
3) What would you change about this presentation?		
4) What suggestions do you have for improvement of this presentation?		
Other comments:		
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Budget Peer Review Form Your Na	me Group Name	
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